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Applications of the Helmholtz equation

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Abstract

In this paper, we are talking about the Helmholtz equation and its physical meaning. Helmholtz equation is used to solve problems in physics such as seismology, electromagnetic radiation, and acoustics. It applies to a wide variety of situations that arise in electromagnetics and acoustics. It is also equivalent to the wave equation assuming a single frequency. In water waves, it arises when we Remove The Depth Dependence. Often there is then a cross over from the study of water waves to the study of scattering problems more generally. Also, if we perform a Cylindrical Eigenfunction Expansion we find that the modes all decay rapidly as distance goes to infinity except for the solutions which satisfy Helmholtz's equation. This means that many asymptotic results in linear water waves can be derived from results in acoustic or electromagnetic scattering.

Introduction

First, we will briefly talk about the Helmholtz equation. The Helmholtz equation, named after Hermann von Helmholtz, is a linear partial differential equation. Associated with the equation are the Laplacian, amplitude, and wavenumber. The Helmholtz equation is also an eigenvalue equation. The Helmholtz differential equation can be solved by separating the variables of everything in polar coordinate systems. The Helmholtz equation often arises in the study of physical problems involving partial differential equations (PDEs) in both space and time. The Helmholtz equation, which represents a time-independent form of the wave equation, results from applying the technique of separation of variables to reduce the complexity of the analysis.

The Helmholtz equation, named after Hermann von Helmholtz, is used in physics and mathematics. This is a partial differential equation, and its mathematical formula is:

$$\nabla^2 W + k^2 W = 0. \tag{1}$$

Here ∇^2 – Lapsasian, k – wavenumber, W – amplitude. The unknown function W is defined by $\mathbf{R}^n$ (in practice, the Helmholtz equation is applied for $n = 1, 2, 3$).

The Helmholtz equation finds application in such concepts of solving physical problems as seismology, acoustics and electromagnetic radiation.

Seismology: The scientific study of earthquakes and their propagating elastic waves is known as seismology. Other study areas are tsunamis (due to environmental effects) and volcanic eruptions (due to seismic sources). There are three types of seismic waves: body waves which have P-waves (primary waves) and S-waves (secondary or shear waves), surface waves and normal waves.

To understand P waves, we have to first look into the basics of seismology and seismic waves. The waves of energy that travel through the earth and cause earthquakes and related phenomena are seismic waves. There are two types of seismic waves:

1) body waves;
2) surface waves.

Body waves are the waves that can travel through the layers of the earth. They are the fastest waves and as a result, the first waves that seismographs can record. Body waves can move through all states of matter including rocks and molten lava. Surface waves can only travel on the surface of the earth. The Helmholtz function is defined as the thermodynamic function of a system which is equal to the difference between the internal energy and the product of the system's temperature and entropy.

The Helmholtz equation was solved by many and the equation was used for solving different shapes. Simeon Denis Poisson used the equation for solving rectangular membrane. Equilateral triangle was solved by Gabriel Lame and Alfred Clebsch used the equation for solving circular membrane. The Helmholtz free energy is defined as the work done which is extracted from the system such that the temperature and volume are constant. Whereas Gibbs free energy is defined as the maximum reversible work which is extracted from the system such that the temperature and pressure are constant.

Fundamental Solutions to the Helmholtz Equation

The Helmholtz equation naturally appears from general conservation laws of physics and can be interpreted as a wave equation for monochromatic waves (wave equation in the frequency domain). The Helmholtz equation can also be derived from the heat conduction equation, Schrodinger equation, telegraph and other wave-type, or evolutionary, equations. The Helmholtz equation is used in the study of stationary oscillating processes. If $k=0$, then equation (1) becomes the Laplace equation.

It is known that the Helmholtz equation in different spaces has different fundamental solutions describing a certain physical process. The Equation (1) has the following fundamental solutions for $n=1,2,3$. [1-3].

For $n=1$, $$W=\frac{ie^{ik|\xi-\eta|}}{2k}$$

For $n=2$, $$W=-\frac{i}{4}H_0^{(1)}\left(k|\xi-\eta|\right)$$

For $n=1$, $$W=\frac{e^{ik|\xi-\eta|}}{4\pi|\xi-\eta|}$$

Finally, for general n, $$W=c_d k^p\frac{H_0^{(1)}\left(k|\xi-\eta|\right)}{2k}$$

where $p=\dfrac{n-2}{2}$ and $c_d=\dfrac{1}{2i(2\pi)^p}$.

These fundamental solutions for different factorizations of the Helmholtz equation are also the same. Using the fundamental solutions of the Helmholtz equation, for various factorizations of the Helmholtz operator, approximate solutions of the Cauchy problem were constructed, in which the solution is presented in explicit form. More precisely, the Carleman matrix is constructed, which allows finding a regularized solution [4-27].

Conclusion

Helmholtz equation is important for various applications. Some of them are as follows:

It is used in seismology which is the scientific study of earthquakes and elastic waves.
Explaining and analyzing natural disaster like Tsunamis.
This equation also plays an important role in Medical imaging.
Through this equation Volcanic eruptions can be explained and predicted.
This equation is also important for the calculation of Electromagnetism

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