



## Analysis of stresses in rotating cylinders of silicon nitride ( $\text{Si}_3\text{N}_4$ ) materials by mathematical modeling

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### Keywords

Rotating cylinder  
Mathematical modeling  
Silicon nitride

### Abstract

Today, there are different studies on the study of the strength values of rotating cylindrical materials. It is known that materials show different properties according to temperatures. The behavior of disc and hollow cylinders, such as shafts, under different temperature conditions is being investigated by other academics. In this study, a rotating cylindrical material with Silicon nitride ( $\text{Si}_3\text{N}_4$ ) material was modeled. The inner and outer semi-diameters of the cylinder are modeled as 40 mm and 120 mm, respectively. The angular velocity of the cylinder is  $w=100$  rad/sec. The stresses obtained are shown with graphs.

### Introduction

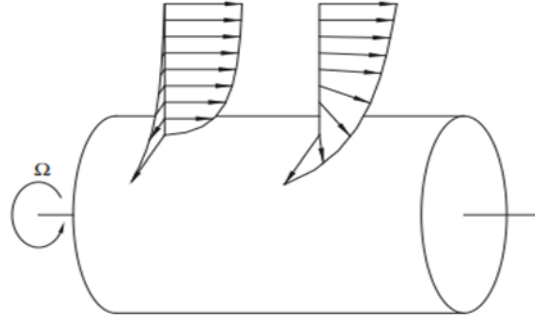
Rotating cylinders can be used in all areas of machine parts. The behavior of rotating cylinders against different temperatures can vary according to temperature and ambient conditions. It is possible that you will come across different studies on this subject in the literature about disks. When the literature review was carried out on this subject; Radial stresses occurring in a disk rotating at a speed of 75 narrow / sec with Grade G4000 material were examined [1]. In different studies, however; Stress analyses were performed on disks with steel wire reinforced thermoplastic composite material, Boron-Carbide ( $\text{B}_4\text{C}$ ) material, Sic/6061 Al Alloy Composite material and disks modeled in different sizes under different temperature conditions. The results obtained were compared with similar studies in the literature [2-6]. In a different study, a training data set was created through artificial intelligence fuzzy logic. In different studies, the stresses occurring in thin rotating cylinders under mechanical load were calculated [7-8].

### Material and Method

The material of the cylinder to be analyzed is Silicon nitride ( $\text{Si}_3\text{N}_4$ ) selected from the appropriate material. Silicon nitride ( $\text{Si}_3\text{N}_4$ ) mechanical properties are given in Table 1. The boundary layer on a rotating cylinder with axial flow is shown in Figure 1.

**Table 1.** Mechanical properties of cylinder material [10]

Modulus of elasticity	w	Density	Cylinder Inner half diameter	Cylinder Outer semi-diameter
470 GPa	100 rad/sn	3170 kg/m <sup>3</sup>	40 mm	120 mm



**Figure 1.** Boundary layer on a rotating cylinder with axial flow [11]

Two-dimensional equilibrium equation in cylindrical coordinates [12];

$$\frac{d(\sigma_r)}{dr} - \frac{1}{r} \frac{d(\tau_{r\theta})}{d\theta} + \frac{(\sigma_r - \sigma_\theta)}{r} + R = 0 \quad (1)$$

For the stress analysis equation in rotating cylinders;

$$r^2 \frac{d^2 F}{dr^2} + r \left[ 1 - r \frac{E'(r)}{E(r)} \frac{dF}{dr} \right] + \left[ \nu(r) \frac{E'(r)}{E(r)} - 1 \right] F = \rho(r) \omega^2 r^3 \left[ r \frac{E'(r)}{E(r)} - \frac{\rho'(r)}{\rho(r)} - 3 - \frac{\nu}{1-\nu} \right] \quad (2)$$

$$\sigma_r = C_1 r^{(n+k-2)/2} + C_2 r^{(n-k-2)/2} + A r^{(2+\gamma)} \quad (3)$$

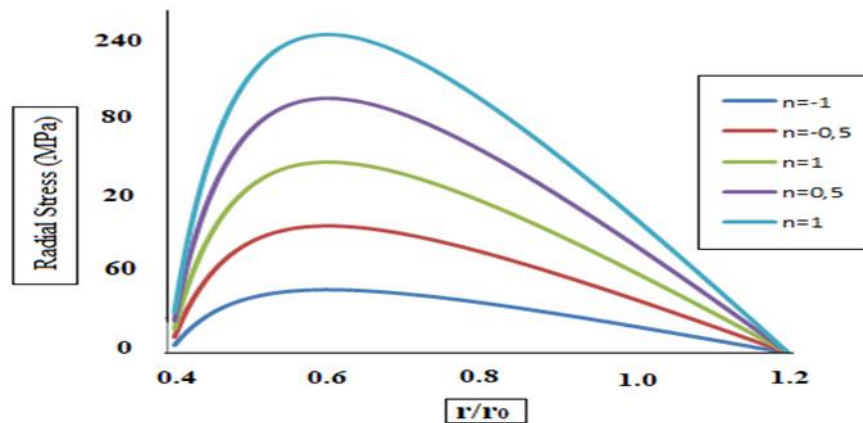
$$\sigma_\theta \text{ (MPa)} = \frac{n+k}{2} C_1 r^{\frac{n+k-2}{2}} + \frac{n-k}{2} C_2 r^{\frac{n-k-2}{2}} + (3+\gamma) A r^{(2+\gamma)} + \rho(r) \omega^2 r^2 \quad (4)$$

According to the formulas above;  $r=et$  transformation is performed,  $E_0$ , modulus of elasticity,  $\rho_0$  density reference value,  $n$  and  $\gamma$  are optional constants.  $C_1$  and  $C_2$  are integral constants. For boundary conditions;

## Results

The radial stress, tangential stress and axial stresses that occur in a cylinder rotating at a speed of  $w=100$  rad/sec with Silicon nitride ( $\text{Si}_3\text{N}_4$ ) material are given in Figure 2-4.

In the previous study, the stresses occurring in the rotating cylinder with Gray Irons (Grade G4000) material were investigated [1]. Similar results have been obtained with this study. Due to the material change, it has been found that the stresses occurring in the cylinder with Silicon nitride ( $\text{Si}_3\text{N}_4$ ) material are higher than Gray Irons (Grade G4000) material.



**Figure 2.** Radial stresses occurring in a cylinder rotating at a speed of  $w=100$  rad/sec

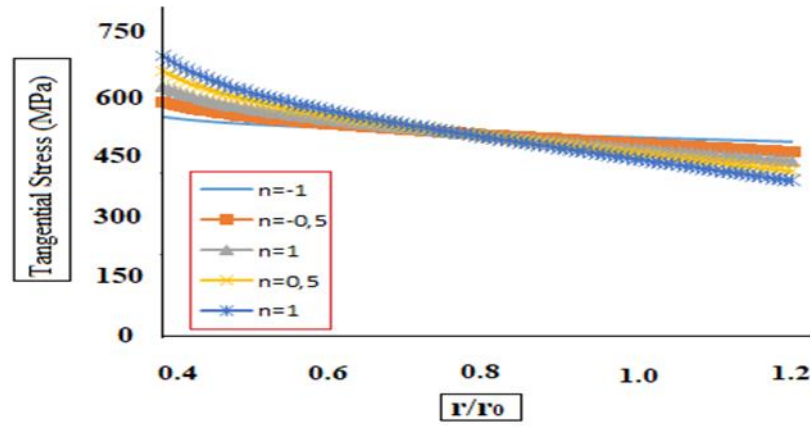


Figure 3. Tangential stresses occurring in a cylinder rotating at a speed of  $w=100$  rad/sec

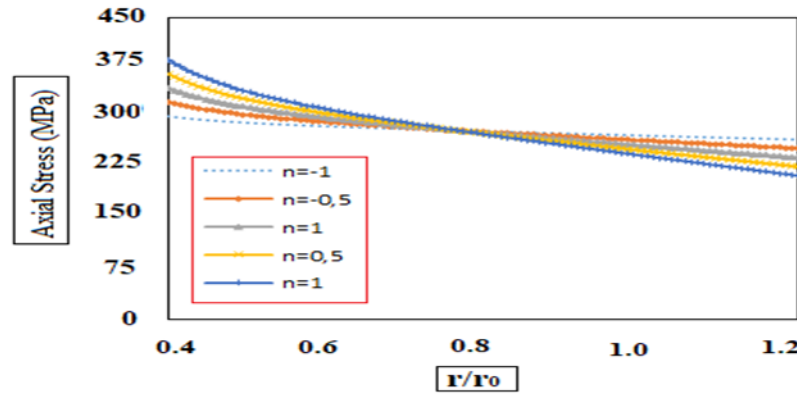


Figure 4. Axial stresses occurring in a cylinder rotating at a speed of  $w=100$  rad/sec

## Conclusion

In this study, the stresses occurring in a cylinder with Silicon nitride ( $\text{Si}_3\text{N}_4$ ) material rotating at a hungry speed of  $w=100$  rad/sec were numerically investigated.

- Maximum radial stresses occurred in the central regions closer to the interior of the cylinder. It was observed that the tangential stress occurring in the cylinder is approximately 320% more than the axial stress.
- Radial stresses in the outermost and innermost parts of the cylinder are zero.
- The effect of stresses is inversely proportional to the increase in the n rating parameter.

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