



## Steady state error and equivalent noise bandwidth analysis of the null-seeker architecture for GPS receivers

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### Keywords

Null-seeker  
GPS  
Tracking  
Noise bandwidth  
Receiver

### Abstract

In this paper is studied the architecture of closed-loop synchronizers. For a GNSS (Global Navigation Satellite System) receiver, the fine estimation of the code delay and Doppler frequency is generally performed by two concatenated null-seekers, the PLL (Phase Lock Loop), and the DLL (Delay Lock Loop). The null-seeker is implemented, tested and analyzed in a software receiver. The noise equivalent bandwidth, integration time and different incoming signal structures are considered for testing and performance evaluation. Different tests have been performed by changing the input signal from a step unit function to a ramp signal and finally to a parabolic shaped signal. The noise-free steady state value of estimation error is evaluated. The type of loop filter defines the tracking capability of the loop. The estimation error must quickly reach zero for a certain input model and any initial error, in the absence of noise.

### Introduction

Synchronizing with the visible satellite signals is an important function of any GNSS receiver [1]. A receiver must first produce a local signal that matches the incoming signal from the satellite before it can give measurements to compute a position, velocity, and timing (PVT) solution. This is done in two stages, namely acquisition and tracking [2]. The objective of the acquisition stage is to find coarse estimates of the Doppler shift and timing offset [3]. An extremely crucial component is the carrier tracking loop, which is utilized to synchronize the local carrier with the incoming signals. Commonly used in the carrier tracking loop, the phase lock loop (PLL) is incredibly fragile, especially in difficult environments [2].

The tracking bandwidth and integration time play an important role on accuracy and dynamic stress tolerance. To reduce the noise and improve accuracy, the tracking bandwidth should be narrow and the integration time long [3]. As a result of the oscillator noise and dynamics on the carrier tracking loop, the bandwidth cannot be reduced arbitrarily. Otherwise, it will cause the phenomena of lock-lose [4]. Due to the complicated environment in the tracking system, accurate models and noise statistics are difficult to be known [2]. For a high sensitivity receiver, no matter whether in acquisition or tracking, the key problem is to extend the coherent time [4]. In [5] the focus is on the process of carrier phase tracking in a scalar PLL. The authors in [6] propose an accurate receiver clock drift estimation method to increase prediction effective time.

In this work is implemented, tested and analyzed a digital synchronization loop architecture in a software receiver. The noise equivalent bandwidth, integration time and different incoming signal structures are considered for testing and performance evaluation.

## Material and Method

The architecture of a closed-loop synchronizer, namely the null-seeker is given in [Figure 1](#) [7]. The input signal  $y[k, \xi]$  is combined with a locally generated reference signal  $x_{ref} = (k, \hat{\xi}[k])$  which has typically the same basic structure as the input signal, apart from the presence of noise and other nuisances. It is characterized by the estimated parameter computed during the previous iteration  $\hat{\xi}[k]$ . The discrimination function is able to transform  $z[k, \xi]$  into a different metric (error signal).  $e_\xi[k, \xi]$  value depends on and is proportional to the estimation error  $e_\xi[k, \xi] \propto \xi - \hat{\xi}[k]$ . A fundamental property is that one of its zeros corresponds to the searched value of the parameter to be estimated. The key operation of a null seeker is to find a zero of its discrimination function (iteratively). The discrimination function  $S(\cdot)$  in (1) can be nonlinear, but it is convenient to study the overall system in its linearity region therefore:  $e_\xi[k, \xi] \approx \beta \cdot (\xi - \hat{\xi}[k])$  where  $\beta$  is the slope of the S-curve in  $\xi - \hat{\xi}[k] = 0$ .

$$e_\xi[k, \xi] = S(\xi - \hat{\xi}[k]) \quad (1)$$

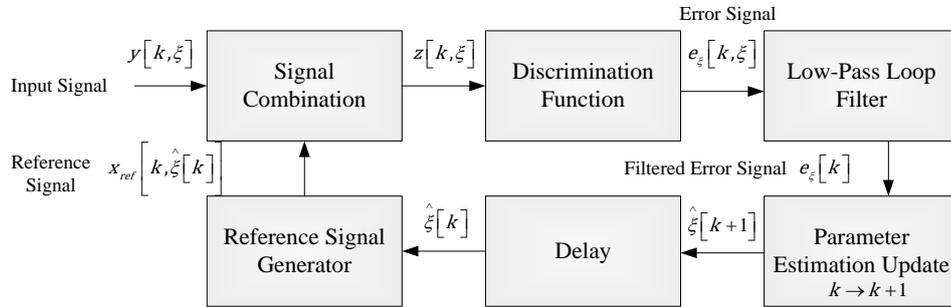
The Low-pass loop filter smoothens the error signal to reduce the contribution of the noise  $w[k]$  and it still preserves the reactivity of the loop to the dynamics of the parameter to be estimated. The new estimated parameter is extracted from the filtered error signal.

$$\hat{\xi}[k+1] = \hat{\xi}[k] + e_\xi[k] \quad (2)$$

The updating rule in (2) followed by the “Delay” block, represents an IIR digital filter, with input  $e_\xi[k]$  and output  $\hat{\xi}[k]$  and its transfer function is  $z\hat{\xi}(z) = \hat{\xi}(z) + e_\xi(z)$ . Transfer function from  $e_\xi(k)$  to  $\hat{\xi}(k)$  can be written in the form:

$$D(z) = \frac{\hat{\xi}(z)}{e_\xi(z)} = \frac{1}{z-1} = \frac{z^{-1}}{1-z^{-1}} \quad (3)$$

As the received signal is noisy, then the error signal contains an additive noise component  $e_\xi[k, \xi] = \beta\delta_\xi[k] + \eta[k]$  where  $\delta_\xi[k] = \xi - \hat{\xi}[k]$  is the instantaneous error and  $\eta[k]$  is a discrete-time random process, white and Gaussian. It is independent from  $\delta_\xi[k]$  and “circulates” within the loop coupled with  $\delta_\xi[k]$ .

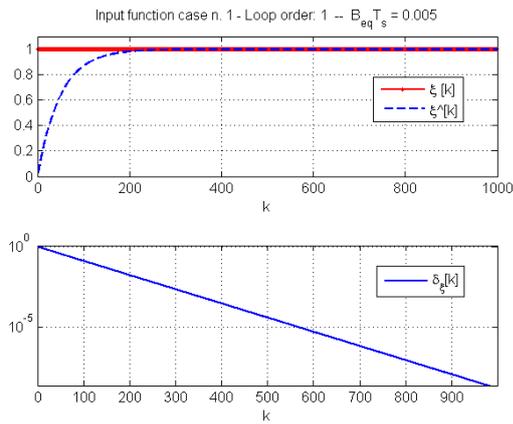


**Figure 1.** Null-seeker architecture

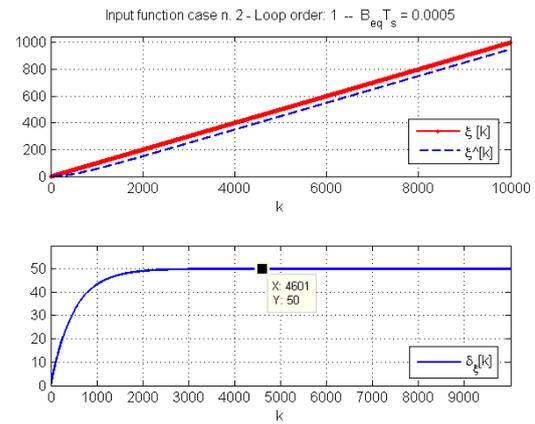
## Simulation and Results

To test the behavior of the null-seeker we have implemented at the software level using Matlab the block diagram in [Fig 1](#). Different tests have been performed by changing the input signal from a step unit function to a ramp signal and finally to a parabolic shaped signal. The other two variable parameters of the simulations are respectively the order of the filter and the product of the equivalent noise bandwidth with the integration time. The steady-state estimation error is evaluated and plotted in [Figure 2 to 5](#).

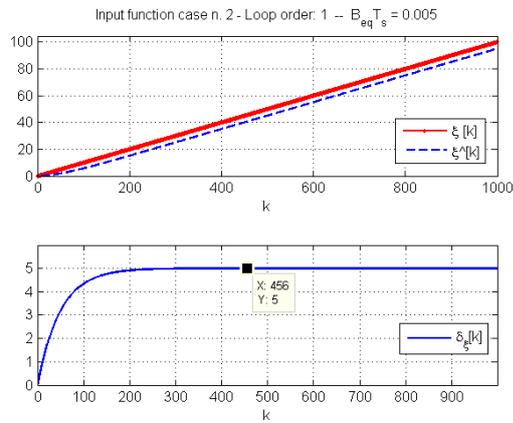
One can observe from [Fig. 3](#) and [Fig. 4](#) that an increase of one order of magnitude of the loop noise equivalent bandwidth (from  $\text{BeqTs} = 0.0005$  to  $\text{BeqTs} = 0.005$ ) decreases  $k$  (from 4601 to 456), and the estimation error decreases with the same order of magnitude (from 50 to 5). So, the integration time is decreased by an order of magnitude. Another important remark that can be derived is that in order to minimize the steady-state error, the noise equivalent bandwidth should be increased. In [Fig. 5](#) can be easily observed that the estimation error  $\delta_\xi[k]$  is unlimited because of the unlimited characteristic of the quadratic input.



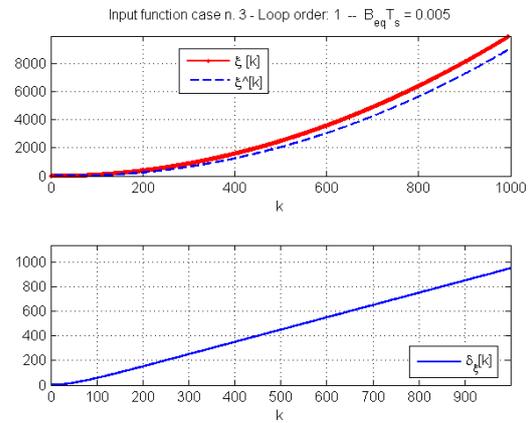
**Figure 2.** Plot of the estimation error  $\delta_{\xi}[k]$  in the logarithmic scale, for step input



**Figure 3.** Plot of the steady state error  $\delta_{\xi}[k]$  for a ramp input



**Figure 4.** Plot of the steady state error  $\delta_{\xi}[k]$  for a ramp input and different  $B_{eq}T_s$



**Figure 5.** Plot of the steady state error  $\delta_{\xi}[k]$  for a parabolic input

## Conclusion

In this article, the architecture of digital synchronizers for GPS receivers was studied. To evaluate the performance of the null-seeker, numerous simulation tests were performed. Three types of input signals were applied: step, ramp and parabolic. From the results we concluded that a higher noise equivalent bandwidth reduces the steady-state error, but this also reduces the integration time.

## References

1. Roncagliolo, P. A., García, J. G., & Muravchik, C. H. (2012). Optimized carrier tracking loop design for real-time high-dynamics GNSS receivers. *International Journal of Navigation and Observation*. Retrieved from <https://doi.org/10.1155/2012/651039>
2. Cheng, Y., & Chang, Q. (2020). A carrier tracking loop using adaptive strong tracking kalman filter in GNSS receivers. *IEEE Communications Letters*. Retrieved from <https://doi.org/10.1109/LCOMM.2020.3018742>
3. Clare, A., Lin, T., & Lachapelle, G. (2017). Effect of GNSS receiver carrier phase tracking loops on earthquake monitoring performance. *Advances in Space Research*. Retrieved from <https://doi.org/10.1016/j.asr.2016.07.002>
4. Cheng, L., Dai, Y., Guo, W., & Zheng, J. (2021). Structure and performance analysis of signal acquisition and doppler tracking in LEO augmented GNSS receiver. *Sensors (Switzerland)*. Retrieved from <https://doi.org/10.3390/s21020525>
5. Curran, J. T. (2015). Enhancing Weak-Signal Carrier Phase Tracking in GNSS Receivers. *International Journal of Navigation and Observation*. Retrieved from <https://doi.org/10.1155/2015/295029>
6. Li, Z., Zhang, T., Qi, F., Tang, H., & Niu, X. (2019). Carrier phase prediction method for GNSS precise positioning in challenging environment. *Advances in Space Research*. Retrieved from <https://doi.org/10.1016/j.asr.2018.12.015>
7. Won, J. H., & Pany, T. (2017). Signal Processing. In *Springer Handbooks*. Retrieved from [https://doi.org/10.1007/978-3-319-42928-1\\_14](https://doi.org/10.1007/978-3-319-42928-1_14)