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The role of algebra and its application in modern sciences

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Abstract

This work deals with the emergence of Algebra and its applications in technical and engineering problems. Algebra becomes important in school when we learn to work with variables, expressions and equations. At first glance, it seems that these are abstract concepts that we will never need in real life. However, in practice, we constantly encounter situations where it is necessary to apply knowledge of algebra to solve problems and make important decisions. One example of algebra being used in everyday life is financial planning. When creating a budget, calculating interest on a loan, or calculating investments, we use algebra to determine optimal strategies and predict future outcomes. Algebra also allows us to solve complex problems related to planning trips or purchasing goods, taking into account our financial capabilities. n addition, algebra has its applications in science and technology. It is the basis for developing algorithms, programming and working with large volumes of data. Thanks to algebra, computers, smartphones, the Internet, and many other technological achievements were created that have become an integral part of our lives.

1. Introduction

Algebra is the branch of mathematics that deals with symbols and the rules for manipulating those symbols. In elementary algebra, these symbols (today written in Latin and Greek letters) represent quantities without fixed values, known as variables. Just as sentences describe relationships between specific words, in algebra equations describe relationships between variables. The history of the emergence of algebra as a science goes back to the distant depths of antiquity. It was then that the basis for generalizing arithmetic operations was laid. This section can be described as a continuation of arithmetic, where numeric values are replaced by alphabetic symbols. We work with elements of sets to generalize the usual operations of addition and subtraction.

It is generally accepted that all the laws of Algebra have long been discovered, but in fact, new directions in this science open up new horizons for the thought of both one person and an entire society. Algebra, in conjunction with absolutely any field of science or art, can provide something new and more advanced. Often it is not necessary to create new theories or areas of algebra; it is enough to use existing data, process them and combine them into one to find a solution to certain problems, of which there will always be a great variety. As an example, let's remember the Google corporation, which began its existence with the implementation of a system for selecting correct information from a large array of data regarding a user request. This system is based on an algebraic algorithm and first worked within the university, and then, being in demand, was developed into the Google search engine. In any other areas of programming, algebra is used no less often. For example, when implementing 3D graphics, a separate direction of algebra is used, which allows using matrices to track the location of each pixel of an object in virtual space. Knowing this, we can conclude that any programming language simply translates the algebra we know into a language understandable to a computer. Just a couple of centuries ago, the construction of cities and especially small settlements required minimal calculations. When building cities, precise calculations are now used. And if during the construction of a city it is often possible to correct small errors, then when constructing such a dangerous facility as a nuclear power plant, errors should be minimal. Here, great responsibility is placed on the correct use of algebraic rules and execution of calculations. Many people sometimes visit amusement parks. Some of them rise tens of meters above the ground. Some of the attractions often take your breath away. However, accurate calculations taking into account various factors make them completely safe and suitable for regular visits. All the facts presented above clearly indicate that Algebra in the modern world is becoming more and more common in every moment of our daily life. All sounds, everything we see and use is built using the science of Algebra.

Algebra is the branch of mathematics in which operations are summarized using numbers, letters, and symbols that represent numbers or other mathematical entities. Algebra is the branch of mathematics that deals with quantity in its most general way. In this sense, it can be noted that the teaching of algebra is dominated by the work "Baldor Algebra" by the Cuban mathematician Aurelio Baldor, who develops and examines all the hypotheses of this science. Etymologically, the word algebra comes from the Arabic language and means "reconstruction" or "reintegration". Algebra comes from the Babylonian and Egyptian civilizations, and before Christ they used this method to solve equations of the first and second order. Later, continuing in ancient Greece, the Greeks used algebra to express equations and theorems, for example: the Pythagorean theorem. The most relevant mathematicians were Archimedes, Heron and Diopanthe. On the other hand, it should be noted that in addition to the book identified above, another book used in Latin America is Mancil Algebra, officially known as "Modern Elementary Algebra", whose authors are Dr. Mario Octavio González Rodríguez and an American mathematician. Dr. Julian Dossy Mancill. At that moment, the students made a mistake in writing the surname, because Mancil should have been written instead of Mancill. In relation to the study of algebra, algebraic expressions are sets of numbers and symbols representing an unknown value known as the unknown or variable. Symbols are related by symbols that indicate operations such as multiplication, addition, subtraction, etc., that must be performed to achieve the result of the variables. In this sense, the terms are differentiated or separated using signs, and when separated by an equal sign, it is called an equation. Elementary algebra develops all the basic concepts. On the other hand, in algebra, a letter represents a value assigned by a person, and therefore it can represent any value. However, when a letter is assigned a specific value in a problem, no value other than that assigned can represent the same problem.

For example: $3x + 5 = 14$. In this case, the value that satisfies the unknown is equal to 3, this value is called the solution or root.

Boolean algebra is (1) or (0) used to represent two states or values that indicate whether a device is open or closed, if it is open because it is conducting, otherwise (closed) because it is not conducting. This system helps to systematically study the behavior of logical components. Logical variables are the basis of programming using a binary system represented by the numbers 1 and 0.

Linear algebra is mainly responsible for the study of vectors, matrices, systems of linear equations. However, this type of algebra section also applies to other fields, such as engineering, computing, and others. Finally, linear algebra dates back to 1843, when Irish mathematician, physicist, and astronomer William Rowan Hamilton coined the term vector and created quaternions. Also with the German mathematician Hermann Grassmann in 1844 when he published the book "Linear Theory of Expansion".

Abstract algebra is a part of mathematics that deals with the study of algebraic structures such as vectors, bodies, rings, groups. This type of algebra can be called modern algebra, in which many of its structures were defined in the 19^{th} century. It was born in order to better understand the complexity of logical statements that are based on mathematics and all natural sciences, which are now used in all areas of mathematics.

In the history of mathematics, many scientists around the world worked on the development of mathematics. It cannot be said that before Al-Khorezmi there was no algebra, in ancient times people solved the simplest algebraic problems, there were techniques for solving individual specific problems, but Al-Khorezmi was the first to introduce algebra as the science of general methods for solving numerical linear and quadratic equations, and gave a classification of these equations, which was essential for “Preliteral” algebra. Historians of science highly appreciate both the scientific and popularization activities of Al-Khorezmi.

The famous historian of science J. Sarton called him "The greatest mathematician of his time and, all things considered, one of the greatest of all time." Al-Khorezmi (full name Abu Abdullah Muhammad ibn Musa Al-Khorezmi) father of Abdullah, Muhammad, son of Musa, native of Khorezm, mathematician, astronomer, historian and geographer of the 9^{th} century. Even the exact dates of his birth and death have not reached us. It is only known that he was born at the end of the eighth century, and died in the second half of the ninth, more precisely after 847. Now it is conventionally accepted to consider the year of his birth to be 783, and the year of death to be 850. The scientist’s homeland was Khorezm, a vast region of Central Asia, which corresponds to the modern Khorezm region of Uzbekistan. Al Khorezmi's diverse scientific interests concerned mathematics, theoretical and practical astronomy, geography and history.

Not all of the works he wrote have survived. In 975-997 he wrote Mafatih al-'Ulum ("Key to the Sciences"), the first Arabic encyclopedia of knowledge that was organized on scientific principles. As a scientist, Al-Khwarizmi becomes famous for his achievements in mathematics. His work on arithmetic was translated into Latin in the 12^{th} century, and although the original is lost, the Latin translation Algoritmi de numero Indorum ("Al-Khwarizmi on Indian numbers") still exists. Its name gave rise to the mathematical term "Arithmetic" [1-3].

2. Material and Method

2.1. The emergence and development of the science of algebra

Algebra is a part of mathematics and it deals with operations on various quantities and solving equations related to these operations. In a broader sense, algebra is understood as a science that studies operations that generalize ordinary operations, such as adding and multiplying numbers on the elements of an arbitrary set. The ages of the three brothers are 30, 20, and 6. After how many years, the age of the eldest will be equal to the sum of the ages of his two brothers. Such equations were known in ancient Egypt in the 2^{nd} millennium BC. But they didn't use letters. At the beginning of the 2^{nd} millennium BC, the ancient Babylonians solved more complex problems. Diophantus, a scientist from Alexandria who lived in the III century, rejects the geometric statement and uses literal expressions. It had symbols for writing negative exponents, negative numbers, and rules for multiplying positive and negative numbers. The further development of algebra was strongly influenced by the algebraic equations studied by Diophantus. From the 6th century, the center of mathematical research moved to India, China, the Middle East and Central Asia. Chinese scientists found a way to successively eliminate the unknowns when finding a solution to a system of linear equations. But algebra, as a special branch of mathematics that describes problems related to solving equations, was formed in the works of scientists of the Middle East and Central Asia. In the 9th century, the Uzbek mathematician and astronomer Muhammad ibn Musa al-Khorazmi (783-850) wrote the work "Al-jabr wal muqabala". In this work, Khorezmi gave a general rule for solving linear equations and divided quadratic equations into classes and showed ways of solving each class. The word al-jabr (restoration) means transferring the negative terms of the equation to its second part by changing its sign. The name of the new science "Algebra" is derived from the same word "Al-jabr". Briefly, looking at the information about al-Khwarazmi, Khorazmi learned to read, write and count in a local religious school, a madrasa. He understood scientific issues better than his teachers, read a lot, worked tirelessly on himself, and did not limit himself to the mandatory textbooks of the madrasa. Khorezm's youth period coincides with the Arab conquest of Khorezm. According to Beruni, the Arab conquerors destroyed the national culture of Khorezm, burned books, took scientists with them, and killed those who did not obey. Probably for this reason, at the end of the 8th century Khorezm came to Baghdad. In the middle of this century, the Abbasids came to the head of the state, and life in the Eastern Arab caliphate began to recover. People of various professions and scientists began to gather in Baghdad. The development of the science corresponds to the period when Harun al-Rashid (786-809) and his son Al-Mahmun were caliphs (813-833). Al-Ma'mun builds "Bayt al-hikmat" (House of the Wise) in Baghdad. It had a good observatory and a rich library. It could be called the Academy of its time. Khorezmi came to Baghdad and engaged in scientific work. Khorezm soon gained fame throughout the Middle East in mathematics, astronomy, geography, history and medicine. He was in charge of scientific works, library, and observatory at the "House of Wise Men". He can be called the first president of the Academy of Sciences. Khorezmi's contribution to mathematics is incomparable. His work entitled "Indian Arithmetic" is dedicated to decimal numbers 0,1,2,...,9. Simplifies them and explains them in Arabic for the first time. These numbers were transferred to the Arabs and then to Europe through Khorezmi's work. The term algorithm in mathematics is also related to the name of Khorezmi, Al-Khorazmi is called al-goritm in Latin and comes from this word. Khorezm was the author of the first mathematical-astronomical tables created in the EAST of the Middle Ages. American Oriental scholar Sorton describes Khorezmi as "one of the greatest mathematicians of all time."

2.2. Works of Al-Khwarizmi

Algebra (Arab. - al-Jabr) is a branch of mathematics. The great Uzbek scientist Abu Abdullah Muhammad ibn Musa al-Khorazmi in his work "Al-jabr wal-muqabala" presented Algebra for the first time in the world. The work was translated into Latin and spread around the world under the name algebra. Algebra refers to substitution, i.e. transfer of terms to the other side of an equation, and substitution means dropping equal terms from both sides of the equation. The main problem of algebra is the study of mathematical operations included in sets. There are mathematical operations that do not look like arithmetic operations at all (for example, there are operations that do not obey the law of permutation or associativity). In Arithmetic, the first four operations on integers are studied. Algebra examines the general properties of these operations applicable to any number and other non-number mathematical objects. In order to achieve the generalization of the results, the values of the quantities are indicated by letters, the rules and laws of actions performed on literal expressions are shown, the rules of changing the form of expressions and solving equations are studied. Omar Khayyom defined Algebra as the science of solving equations. His definition remained valid until the end of the 18th century. Algebra has expanded in new directions in the subsequent period, but it has retained its importance as a general science of operations. Ancient Egyptians solved more complex problems (arithmetic and geometric progressions). The definition of problems and their solution were given verbally only for a few examples. These examples show that general methods for solving 1st- and 2nd-level problems in terms of form are accumulating. Greek geometry stood out. Here, geometrical checks were conducted by logic in such a way that every thought was not left without proof. As a result of the strong influence of geometric considerations, problems of arithmetic and algebra geom. was explained with the language.

For example, a quantity was considered as a length, and the product of two quantities was considered as the face of a rectangle. Present tense mat. Calling the multiplication of a quantity by itself a "square" is an example of the survival of the geometric language. Mathematicians of Turkestan made a great contribution to the completion, generalization and development of the results achieved by the Greeks. Mathematical scientists of Turkestan have successfully solved calculation of roots, methods of approximate solution of a number of equations, giving the general formula of Newton's binomial in words. In the 9th and 10th centuries, Turkestan becomes a major scientific center. During this period, al-Khorazmi and Abu Rayhan Beruni lived and became famous in the world with their great scientific works. In 1074, another book of Umar Khayyam called "al-Jabr" showed ways to solve linear and quadratic equations, find the roots of third-order equations by geometric method and solve them. In the works of Ibn Sina, the solutions of the problems of arithmetic and algebra, which were of special importance for that time, were given. In his work on mathematics, particularly Algebra and Arithmetic, the operations of squaring and cubing numbers were investigated. From the history of the ancient world until the time of al-Khwarizmi, mathematics was not separated into knowledge such as algebra and arithmetic. Algebra became a separate branch of mathematics only from the time of al-Khwarizmi. The establishment of the famous Ulugbek observatory in Samarkand in the 15th century led to the development of astronomy as well as the development of mathematics. For the development of algebra, it was necessary to find convenient symbols instead of expressing operations in words. This work went very slowly: Egyptians used a special symbol for fractions.

Al-Khorezmi was the first to introduce the Arabs, and through them the Europeans, to the “Arabic numerals” invented by the Indians, he was the founder of algebra, his name in Latin translations became the term “algorithm”, he was the author of some of the first Arabic astronomical, geographical and historical treatises. It is difficult to overestimate the merits of al-Khorezmi to world science. He was the founder of a powerful cohort of scientists of the medieval East, whose activities built a bridge from ancient science to the science of the Renaissance. In particular, F. Engels, speaking about the formation of natural science as a real science, emphasized that it developed in the Middle Ages precisely in the East: “The beginnings of an accurate study of nature began to develop among the Greeks of the Alexandrian period, and then, in the Middle Ages, among the Arabs. Real natural science begins only in the second half of the 15th century...” x. Of course, the concept of “Arabs” here is not only an ethnonym; it applies to all medieval scientists of the East who wrote in Arabic, the international language of science of that time. Al-Khwarizmi introduced the Arabic-speaking scientific world to “Indian counting,” a decimal positional numbering system that was later adopted by Europeans and called “Arabic numerals.” His algebraic treatise marked the beginning of the spread of algebra in Europe and gave the name to this branch of mathematics. The very name of the scientist al-Khorezmi, indicating his origin from the Central Asian state of Khorezm, was transformed in the mouths of European scientists into the word Algor ithmus, which initially denoted the decimal positional system and arithmetic operations in this system, and then turned into the modern term “algorithm”, or “algorithm". Al-Khwarizmi's works on astronomy played an exceptional role in the history of this science. Al-Khorezmi owned some of the first astronomical tables, which, like his mathematical works, became early known in Evpone from Latin translations and had a great influence on the development of world astronomy. Al-Khorezmi's geographical work was the first in its field in Central Asia, the Near and Middle East and paved the way for the development of geography in these regions. Al-Khorezmi was not alone in his activities. Far from his homeland, in Baghdad, his great compatriots worked with him - Central Asian scientists who found themselves together due to common historical circumstances and their destinies. The fate of information about the lives of people of the distant past, as well as the fate of these people themselves, are not the same. If it happened that a student of Abu Ali ibn Sina (Avicenna) wrote down a fairly detailed biography of him from the words of his teacher, and then supplemented it, and if the life path of Abu Rayhan Biruni can be traced quite clearly on the basis of autobiographical facts scattered in his works, and besides While later authors report a lot about both Ibn Sina and Biruni, then al-Khorezmi did not have his own biographers and says nothing about himself. Fragmentary information about him in later sources is extremely scarce. The earliest and subsequently substantially unenriched information about al-Khorezmi is contained in the first collection of biographies of scientists in Arabic-language literature, compiled in the 19th century. Ibn al-Nadim and called by him simply “List” or “List” (“al-Fihrist”). This biography is so short that it can be given here in full:

“Al-Khwarizmi. His name is Muhammad ibn Musa, and his origin is from Khorezm. Khorezm can also be called the country of ancient science in Central Asia and the country of advanced science in this region in all periods of its ancient and early medieval history. The stable local scientific traditions that developed here in pre-Islamic times, enriched by cultural communication with other peoples, were one of the most important prerequisites for the formation in this country of such great thinkers of world significance as al-Khorezmi and Biruni.

This is especially true for astronomy and closely related mathematics. These sciences were vital, first of all, for the development of the basis of the Khorezm economy - irrigation. They served the purpose of determining periods of high water and the directions of canals being built, adjusting water intake in them, measuring and calculating the areas of irrigated land. The maintenance of the agricultural calendar was based on them. Without mathematics, it was impossible to develop construction, trade and solve a range of property issues. It is not only we today who judge the practical significance of these sciences for the distant past from the height of the current level of knowledge. The scientists of Khorezm themselves clearly imagined and deeply understood this [4-7].

It is considered established that Al-Khwarizmi was the author of 9 works:
1. Book about Indian arithmetic (or Book about Indian counting);
2. A short book on the calculus of algebra and almukabala;
3. Astronomical tables (zij);
4. Book of pictures of the Earth;
5. Book about building an astrolabe;
6. A book about actions using an astrolabe;
7. Book about sundial;
8. Treatise on the definition of the era of the Jews and their holidays;
9. History book.

Of these books, only seven have reached us in the form of texts either by Al-Khwarizmi himself or his Arab commentators, or in translations into Latin. Al-Khwarizmi's work on arithmetic played a vital role in the history of mathematics, and although its original Arabic text is lost, its contents are known from a 12th-century Latin translation, the only manuscript of which is kept in Cambridge. This work provides the first systematic presentation of arithmetic based on the decimal positional number system. The translation begins with the words "Dixit Algorizmi" (said Algorizmi).

In Latin transcription, the name Al-Khorezmi sounded like Algorizmi or Algorizmus, and since the essay on arithmetic was very popular in Europe, the author's name became a household name - medieval European mathematicians called arithmetic based on the decimal positional number system. Later, this was the name given to any system of calculations according to a certain rule; now this term means a prescription that specifies a calculation process, starting with arbitrary initial data and aimed at obtaining a result completely determined by these initial data [4].

The algebraic book of Al-Khorezmi (Kitab mukhtasab al-jabr and wa-l-mukabala) consists of two parts - theoretical (theory of solving linear and quadratic equations, some questions of geometry) and practical (application of algebraic methods in solving household, trade and legal tasks - division of inheritance, drawing up wills, division of property, various transactions, measuring land, building canals). The word al-jabr (replenishment) meant transferring the negative term from one side of the equation to another, and it was from this term that the modern word "algebra" arose. Al-muqabala (contrast) - reduction of equal terms in both sides of the equation. The doctrine of linear and quadratic equations, inherited from Eastern mathematicians, became the basis for the development of algebra in Europe. The geometric part of the treatise is devoted mainly to measuring the areas and volumes of geometric figures: triangle, square, rhombus, parallelogram called rhomboid, circle, segment of a circle, quadrangle with different sides and angles, parallelepiped, circular cylinder, prism and cone [8-10].

2.3. Algebra in technical and engineering problems

One of the main applications of algebra in engineering is solving systems of linear equations. For example, in structural mechanics such systems can arise when calculating stresses and strains in building elements. To solve a system of linear equations, it is necessary to apply matrix methods that are based on algebra. Algebra is used in calculating optimal parameters in various problems. For example, when designing electrical circuits, it is necessary to determine the optimal values of resistance, inductance, and capacitance to achieve the best electrical performance. To solve such problems, optimization methods are used that are based on algebra and matrix operations. Algebra is also used to analyze trends and predict future trends in various fields. For example, in finance, analysts may use algebra to build models that predict changes in stock prices or currencies. Similarly, in engineering, algebra can be used to analyze and predict various parameters such as temperature, pressure and speed. Many problems in engineering and technology involve processing large amounts of data and statistical analysis. For example, when analyzing the results of tests or experiments, it is necessary to perform statistical analysis of data that may contain thousands or even millions of numbers. To solve such problems, linear and nonlinear regression methods, which are based on algebra, are often used.

The development of technology in the modern world forces engineers to solve a wide variety of problems: calculation of structures, design of machines, engines and objects such as airplanes, diesel locomotives, spaceships, etc. The solution to these problems has a common scientific basis due to the fact that these problems contain questions that require the study of the laws of motion or equilibrium of bodies. In mechanics, interaction with vector objects such as speed, acceleration, force, etc. is quite often carried out. And most often you have to work not with individual vectors, but to create a system. For example, the sum of forces acting on a rigid body can be represented as a single object - a set of vectors showing these forces. As a result, problems of this kind raise the need to include in the usual laws of vector algebra certain laws that reflect the transformation of sets of vector systems. The formation of such laws becomes impossible without restrictions on the sets under consideration. These restrictions must be quite extensive, since they must cover important sets of vectors that appear in problems of mechanics and are sufficiently rigid that it is possible to establish general laws for the class of sets they distinguish. In order to consider the relationship between vector algebra and mechanics, we need to give definitions of a vector in mathematics and mechanics. In mathematics, a vector is a directed segment; in mechanics, it is a quantity that is characterized by direction, numerical value (modulus) and point of application.

Algebra is the study of the properties and structures of algebraic objects such as numbers, variables, operations, equations, and more. It is based on algebraic laws that describe the behavior of these objects when performing operations. One of the important concepts of algebra is an algebraic system, which consists of many elements and algebraic operations on them. Algebra has a wide range of applications in various areas of mathematics and its application is closely related to solving mathematical problems and problems. Algebra is used to analyze data and solve statistical problems. For example, when solving linear regression problems, algebraic methods allow you to determine the parameters of the model in accordance with the observed data. In combinatorics, algebraic methods are used to solve problems involving counting combinatorial objects, such as arrangements, combinations, and permutations. Algebra allows you to simplify and structure calculations and solve problems. Algebra is used in cryptography, the science that studies methods of protecting information. Cryptographic algorithms are based on mathematical principles, such as algebraic groups and fields, to ensure the security of transmitted data. Algebra is the basis for formulating and solving physical and engineering problems. It is used to model and describe various physical phenomena such as body motion, electromagnetism and fluid mechanics. Algebra plays an important role in the development of other sciences. Functional analysis, the branch of mathematics that studies spaces of functions and operations on them, is heavily dependent on algebra. In particular, algebraic structures such as Banach algebras and operator algebras play a key role in the analysis of linear operators. Graph theory uses algebraic methods to study the properties and structures of graphs. For example, algebra is used to solve minimum spanning tree, shortest path, and other combinatorial problems. Algebraic methods are used to analyze and design algorithms. Algebra notation allows you to describe and formalize the steps of an algorithm, simplifying their implementation and analysis.

One important application of algebra is in financial mathematics. With its help, we can analyze and manage financial flows, solve problems of investment, insurance and lending. Algebra is also an integral part of statistics, economics and accounting. Another important aspect of algebra is its application in engineering and technology. She assists in algorithm development, programming, complex system analysis, and design. Without algebra, it would not be possible to create modern computers, programs, mobile devices and other technical innovations. Thus, algebra plays a key role in real life. It is an integral part of our world and allows us to solve various problems, analyze and present data, develop new technologies and make discoveries in science. Therefore, understanding and mastering algebra are not only of theoretical importance, but also of practical significance in everyday life. The first and most obvious use of algebra is in solving mathematical problems. Algebraic methods allow us to analyze and solve various problems using formulas, equations and matrices. For example, algebra helps us solve problems in finance, calculating the cost of goods, taxes, loans and investments. Additionally, algebra plays an important role in physics and engineering. It allows us to describe and predict various physical phenomena, calculate the trajectories of bodies, and simulate electrical and mechanical systems. Without algebra, it is impossible to imagine modern technologies and developments in the field of engineering. An equally significant application of algebra is automation and data analysis. Algebraic methods are used in computer algorithms to process and analyze data, for example in the development of machine learning and artificial intelligence algorithms. Without algebra, it would be impossible to cope with tasks such as processing large amounts of data and solving complex optimization problems. Finally, algebra is important in every person's daily life. For example, we use algebraic methods when planning and managing our finances, when solving a scheduling problem, when calculating distance and time when traveling, when calculating shares and discounts in a store. Thus, algebra is an integral part of our daily life and plays an important role in solving various problems. Understanding algebra and its application helps us better understand and analyze the world around us, make informed decisions, and develop our thinking skills. Algebra is also widely used in financial planning. With it, we can develop models that help us forecast financial results, conduct sensitivity analyses, determine optimal strategies, and make informed financial decisions. Additionally, algebra is used in financial analytics to process and analyze large volumes of financial data. Using algebra, we can conduct regression analysis, empirical and modeling studies, and predict future trends in financial markets. From the above it is clear that algebra is an integral part of financial calculations and analysis. Thanks to algebra, we can overcome complex financial issues, make informed financial decisions, and achieve better results in financial management.

One important area where algebra is applied is in electrical and electronic engineering. Algebra allows you to analyze and synthesize electrical circuits, solve equations related to current and voltage, and also model and simulate the operation of various electronic devices. In mechanics, algebra is used to solve problems involving the motion of bodies. Using algebra, you can derive equations that describe the movement of a body, determine its speed, acceleration, and predict its movement in the future. Algebra is also applicable in the fields of construction and architecture. With its help, you can solve problems related to the calculation of building structures, determination of volumes and areas, analysis of forces and loads. Due to the development of computer technology and the use of software, algebra is becoming an integral part of the work of engineers and technicians. With its help, you can create mathematical models and solve complex problems, conduct data analysis and present research results. The basic principle of algebra is working with symbols and expressions. Knowledge of algebra allows us to analyze and solve complex problems, especially in fields related to science, economics and finance. In the business world, algebra is used to calculate the cost of goods, production planning, and analytical forecasting. Knowledge of algebra will help entrepreneurs optimize their activities and make informed decisions. In engineering, algebra is also an integral part. It is used for complex system modeling, power and electrical circuit calculations, data analysis, and hypothesis testing. Without algebra, it would not be possible to create innovative technologies and develop new products.

2.4. Applications of linear algebra in programming

We'll tell you what practical problems linear algebra helps solve, and how abstract matrices and linear spaces can be useful in developing real software. Linear algebra is a branch of mathematics devoted to the study of systems of linear equations, linear mappings, and vector spaces. The main tools of linear algebra are matrices, determinants and dual spaces. Matrices and determinants (determinants) are used in linear algebra to solve systems of linear equations and perform operations on vectors. They also play an important role in the theory of linear mappings and in the study of the properties of vector spaces and subspaces. Dual spaces are another important concept in linear algebra. They represent a set of linear functionals that can be used to describe the properties of vector spaces.

Linear algebra has a wide range of methods that help:

1. Process and analyze large volumes of data;
2. Develop effective encryption algorithms;
3. Create realistic graphics;
4. Recognize patterns;
5. Simulate any objects, events, systems;
6. Predict events based on data from the past;
7. Optimize various systems and mechanisms.

This has made linear algebra an important mathematical tool in fields ranging from economics, physics, biology and engineering to game development and machine learning. Let's take a closer look at some ways to use linear algebra in application software development. Linear algebra tools allow you to perform various operations with data, including finding dependencies between variables, data dimensionality reduction, cluster analysis and image processing. In machine learning, matrix operations are used in classification, regression, and clustering algorithms.

Linear Regression – This method is used to analyze the relationship between two variables. Linear regression uses matrix operations to find a linear relationship between variables and predict the values of one variable based on another.

Logistic regression - when building a model, the least squares method is used, and when using the maximum likelihood method, matrix multiplication and inverse matrix are used to find the optimal parameter values. In addition, linear algebra is used to calculate the gradient of the loss function and to update the model parameters during the training process.

Principal component method is used to reduce the dimensionality of data. Matrix operations are used to find linear combinations of variables that preserve the information in the data as much as possible. This allows you to reduce the dimensionality of the data and simplify their analysis.

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Cluster analysis is a method that allows you to group data by similarity. Here, linear algebra is used to calculate distances between objects and to find clusters that are maximally different from each other.

Neural networks - when creating and optimizing models, linear algebra is used to calculate the weights and biases of neurons, and during the training process they are used to find errors.

Convolutional neural networks - here linear algebra is used to process images and find features that are used to classify and recognize objects.

Support Vector Machine – used to find the boundary that separates two classes. The method is used for pattern recognition, determining the category of an object and other classification tasks.

Linear discriminant analysis - used to find the best linear combinations of variables that can separate two or more categories (groups). Used in machine learning to classify data.

Linear algebra is one of the most important tools in game development: its applications range from 3D graphics and physical modeling to animation and artificial intelligence.

3D graphics – linear algebra is used to describe and transform objects in 3D space. It helps determine the location and orientation of objects in space, configure cameras and light sources, and also perform object transformations - scaling, rotating and moving.

Physical modeling - here linear algebra helps to realistically simulate the behavior of objects. For example, linear algebra is used for:

1. Determining the trajectory of motion of objects, taking into account their mass, forces acting on them and acceleration;
2. Collision calculation;
3. Realistic simulation of water, snow, wind, fog, etc.

Animation - Linear algebra helps determine the position and orientation of objects in various frames of animation, and also provides the calculation of natural motion curves for objects. This allows you to animate characters realistically, determine camera directions, and create visual effects.

Artificial Intelligence – Linear algebra is involved in processing and analyzing data that is used in AI algorithms. Such algorithms can determine the behavior of opponents, create new routes and obstacles, and plan the optimal campaign strategy.

Linear algebra is the standard language for quantum computing. This is because quantum mechanics describes the states of quantum systems as vectors in linear Hilbert space. Quantum algorithms use linear algebra to manipulate quantum states, process data, and solve computational problems. In particular, operations on quantum states that describe the interaction of quantum systems are expressed as linear operators (unitary matrices) on Hilbert spaces. Quantum computing is based on the use of quantum gates, which are linear operators applied to qubits (analogous to bits in classical computing). They are usually represented by matrices that describe various quantum gates - Hadamard transform, phase shift, Pauli Y gate, etc. These gates find application in algorithms for quantum Fourier transform, quantum Grover search, quantum key distribution and others. In addition, linear algebra is used to work with quantum channels, which describe the transmission of quantum states through a noisy channel, as well as to solve quantum optimization problems, where the optimization problem is represented as minimizing or maximizing a quantum linear function, which is expressed as a linear combination of quantum valves and operators.

Linear algebra is widely used in creating 3D modeling software (ZBrush, 3ds Max, Blender, Maya and others). Mathematical processing - using linear algebra, you can create mathematical models of any objects. This allows you to perform all the necessary calculations - calculating the volume and area of models, its surface, mass distribution, etc. Transformation of objects - linear algebra is used to transform three-dimensional objects, move, scale and rotate models. This is done using matrices that contain information about changes in the location, size and orientation of the model.

Lighting and Shadows - Linear algebra methods are used to calculate lighting effects and shadows on 3D models. This includes calculating the direction of light, the distance to objects, and determining the positions of objects in three-dimensional space.

Animation of objects - during the animation process, linear algebra helps to implement object and camera movement, changing shape or texture, directing light, switching between scenes. Surface Normalization – Linear algebra is used to adjust and normalize the surface of a 3D model. This allows you to create more realistic textures and surfaces in your models. Collision calculation - linear algebra allows you to implement physically correct collisions and mechanics - the movement and interaction of objects [11-14].

3. Conclusion

Algebra is a fundamental tool for solving mathematical problems and problems. Its applications cover a wide range of fields, including data analysis, combinatorics, cryptography, physics, and more. In addition, algebra plays an important role in the development of other sciences, where algebraic methods are used to study the structure and properties of objects and develop algorithms.

Astronomy occupied a leading place among the exact sciences in the medieval East as one of the most necessary sciences in practice; it was impossible to do without it either in irrigated agriculture or in sea and land trade. By the 9th century include the first independent works on astronomy in Arabic, a special place among them was occupied by ziji collections of astronomical and trigonometric tables (at that time trigonometry was part of astronomy), with the help of these tables the positions of the luminaries on the celestial sphere, solar and lunar eclipses were calculated, they served and for measuring time. Among the first zijs is the zij of Al-Khorezmi, which began with a section on chronology and calendar - this was very important for practical astronomy, since different peoples used different calendars at different times, and dating is important when making observations. There were lunar, solar and lunisolar calendars, and the beginning of chronology in various systems referred to an arbitrarily chosen event. This led to many different eras; different peoples dated the same event differently, in accordance with the era they adopted. Al-Khwarizmi described the Arabic lunar calendar, the Julian calendar - the calendar of the "rums" (Romans and Byzantines). Humanity owes all the greatest discoveries that have given the world geniuses to mathematics. Today everyone needs mathematics. This is the call of the times, an urgent necessity of life. Accelerating scientific and technological progress is impossible without managing the volume of mathematical knowledge at all levels, starting primarily from high school, academic lyceums, colleges and higher educational institutions.

We are faced with financial decisions every day, whether it's budgeting, calculating mortgage payments, investing, or managing debt. Algebra allows us to solve complex problems efficiently and accurately. Interestingly, the history of the emergence of algebra is not limited to Europe and having contact with her Arab civilization. Thus, the significant results achieved in the Indian science of mathematics. In particular, they introduced the concept of "zero", which later came across the Arab world to Europe and has been used by scientists. The Chinese are quite independently, since the dawn of our era, have learned to solve the equations of the first degree. They were known to the irrational and negative numbers.

Algebra is one of the most important branches of mathematics, which is widely used in engineering and technology. In this article, we looked at several examples of problems that can be solved using algebra, and showed that without its help, engineers and technologists would be unable to solve many complex problems. Today, algebra continues to evolve and find new applications in various fields such as physics, economics, computer science, and game theory. Its history shows how humanity used mathematics to solve practical problems and create new knowledge that became the foundation for the development of science and technology [15].

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Author contributions

Davron Aslonqulovich Juraev: Conceptualization, Methodology, Software, Writing-Reviewing and Editing. **Murot Nashvandovich Bozorov:** Data curation, Writing-Original draft preparation, Software, Validation.

Conflicts of interest

The authors declare no conflicts of interest.

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