

# Calculation of Centering Elements by Methods Other Than the Classical Method 

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## Keywords

Centering elements, Reduction of off-center observations to the center, Calculation of centering elements,


#### Abstract

In cartography, when it is necessary to measure the angle at the center of the minaret, on which no tools can be installed, a point marked on the minaret balcony is used as the off-center point. The calculation of the basic elements required to reduce the angle measurements made at this off-center point to the center point is called "Calculation of Centering Elements". In the local coordinate system created with the help of two points established on the land near the minaret in the classical method, the centering elements can be calculated after the coordinates of the center point and the off-center point are found. It is possible to calculate the centering elements with other methods besides the classical method. As a matter of fact, there is also a study in the past that provides convenience in calculation. In this study; taking into account the other study on this subject, another method that does not require coordinate calculation is explained. Numerical applications related to the subject have been made in such a way as to reset the effects of rounding errors. The results obtained with other methods and the classical method were examined. At the end of the study, the findings and opinions are stated.


## 1. INTRODUCTION

In cartography, when it is necessary to measure the angle at the center of the minaret, on which no tools can be installed, a point marked on the minaret balcony is used as the off-center point. The calculation of the basic elements required to reduce the angle measurements made at this off-center point to the center point is called "Calculation of Centering Elements". In order to reduce the measurements made at the off-center point to the center point, the so-called centering elements must be calculated. In order to make this calculation, two auxiliary points (A, B) are established on the ground near the minaret (Figure 1).

In the classical method, a local coordinate system is created with the help of these points, one of these points is accepted as the starting point, and the coordinates of the center point ( $Z$ ) and the off-center point (D) are calculated. After these calculations, the centering elements ( $\mathrm{DZ}=\mathrm{e}, \alpha \mathrm{Z}$ ) are calculated. Instead of this method, which requires a long calculation process, another method has been developed that provides convenience in the past (Allan et al. 1968; Kıran 1983). Today, it is possible to calculate the centering elements by another method.

In this study; Taking into account the other study on this subject, another method that does not require coordinate calculation is explained (Figure 2). Practices related to the subject were made, and the findings and opinions obtained as a result of the study were stated.


Figure 1. Centering elements according to the classical method


Figure 2. According to the first method centering elements

## 2. CALCULATION METHODS of CENTERING ELEMENTS OTHER THAN the CLASSICAL METHOD

### 2.1. First Method

The basis of this method is to mark the off-center point on the minaret balcony on the direction connecting the auxiliary point on the ground to the central point (Kıran 1983; Wolf and Ghilani 2008).

In Figure 2;
Z: The center point in the minaret realm,
D: The off-center point marked on the minaret balcony,
$A$ and B: Auxiliary points marked on the land near the minaret,
$\mathrm{DZ}=\mathrm{e}, \alpha \mathrm{z}$ : Indicates the centering elements.
Angles $\mathrm{ZZ}=\mathrm{TD}, \beta \mathrm{Z}, \beta \mathrm{D}$ and edge AB are measured from points $A$ and $B$ in the field.

In addition, horizontal distances AD and BD can be measured with an electronic tachometer.

From Figure 2, angle $\varphi$ and sides $\mathrm{AZ}, \mathrm{BZ}$ and DZ in triangle AZB are obtained from the following relations.
$\varphi=\gamma_{\mathrm{Z}}+\beta_{\mathrm{D}}$
$\mathrm{AZ}=\frac{A B * \sin \beta \mathrm{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}$
$\mathrm{BZ}=\frac{A B * \sin \delta \mathrm{Z},}{\sin \left(\gamma_{Z+\beta}\right)}$
DZ=AZ-AD

Since the direction measurements are made from the $D$ point to the surrounding triangulation points, $A$ and $B$ points, the unknown $\alpha z$ is obtained from the following relations (Kıran 1983).
$\alpha_{z}=\alpha_{A}+200$
$\alpha_{z}=\alpha_{B}-\varphi$
To make this method even easier, the following application can be made. With the electronic tachometer instruments produced today, it is possible to orientate
from one point to another with a certain directional angle (URL_1). By making use of this feature of the instrument, if the direction angle of $\alpha \mathrm{A}=200.0000$ is viewed as the direction angle to point A while observing point A from the off-center point D , then the direction angle of the Z center point becomes $\alpha z=0.0000$. Relation (6) is used as a control.

After obtaining the $\alpha z$ and DZ centering elements, the rules specified in the reduction of off-centre observations to the center are applied to reduce the angles measured at the D point to the Z point. (Atasoy 2014; Aydın 1974; İnce et al. 2021; Uren and Price 1986; Özbenli and Tüdeş 1989; Şerbetçi and Atasoy 2000).

### 2.2. Second Method

The basis of this method is calculating the centering elements from the triangles formed by tangent and cosine relations without making coordinate calculations. By making observations from the auxiliary point $A$ to the points $B, Z, D$, the angles $0 Z$, $D_{D}$ and the edge $A B$ are measured. $\beta \mathrm{Z}, \beta \mathrm{D}$ angles are measured by observing from auxiliary point $B$ to points $\mathrm{Z}, \mathrm{D}, \mathrm{A}$. The sides of $\mathrm{AZ}, \mathrm{AD}, \mathrm{BZ}$, BD are obtained from the following equations by applying the sine relation in triangles ABZ and ABD (Figure 3).


Figure 3. Observations from the auxiliary point $A$ and $B$ to the points $B, Z, D$
$\mathrm{AZ}=\frac{A B * \sin \beta \mathrm{Z},}{\sin \left(\gamma_{Z}+\beta_{Z}\right)}$
$\mathrm{BZ}=\frac{A B * \sin \gamma_{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}$
$\mathrm{AD}=\frac{A B * \sin \beta \mathrm{D},}{\sin \left(\beta_{D}+\gamma_{D}\right)}$
$\mathrm{BD}=\frac{A B * \sin \delta \mathrm{D},}{\sin \left(\beta_{D}+\gamma_{D}\right)}$
In Figure 3, $a=A D, b=A Z, c=B Z, d=B D, d \gamma=\gamma_{D}-\gamma_{Z,} d \beta=$ $\beta_{z}-\beta_{D}$ are abbreviated. In triangles AZD and BZD, the cosine relation for DZ is written.
$\mathrm{DZ}=\sqrt{\left(a^{2}+b^{2}-2 * a * b * \cos d \gamma\right)}$
$\mathrm{DZ}=\sqrt{\left(c^{2}+d-2 * c * d * \cos d \beta\right)}$
In Figure 3, the following relation is written for $\frac{\varphi_{1}+\psi_{1}}{2}$ in triangle AZD
$\frac{\varphi_{1}+\psi_{1}}{2}=\frac{200-d \gamma}{2}$
and the tangent relation for $\frac{\varphi_{1}-\psi_{1}}{2}$ is written.
$\frac{\varphi_{1}-\psi_{1}}{2}=\arctan \left((a-b) * \tan \left(\frac{\varphi_{1}+\psi_{1}}{2}\right) /(a+b)\right.$
Considering equations (13) and (14), $\varphi_{1}$ and $\psi_{1}$ are obtained from the following relations.
$\varphi_{1}=\frac{\varphi_{1}+\psi_{1}}{2}+\frac{\varphi_{1}-\psi_{1}}{2}$
$\psi_{1}=\frac{\varphi_{1}+\psi_{1}}{2}-\frac{\varphi_{1}-\psi_{1}}{2}$
Similarly, in Figure 3, the following equations are written for $\frac{\varphi_{2}+\psi_{2}}{2}$ and $\frac{\varphi_{2}-\psi_{2}}{2}$ in triangle BZD.
$\frac{\varphi_{2}+\psi_{2}}{2}=\frac{200-d \beta}{2}$
$\frac{\varphi_{2}-\psi_{2}}{2}=\arctan \left((c-d) * \tan \left(\frac{\varphi_{2}+\psi_{2}}{2}\right) /(c+d)\right.$
Considering the equations (17) and (18), $\varphi_{2}$ and $\psi_{2}$ are obtained from the following relations.
$\varphi_{2}=\frac{\varphi_{2}+\psi_{2}}{2}+\frac{\varphi_{2}-\psi_{2}}{2}$
$\psi 2=\frac{\varphi_{2}+\psi_{2}}{2}-\frac{\varphi_{2}-\psi_{2}}{2}$

Considering the direction angles $\alpha_{A}$ and $\alpha_{B}$ in Figure $3, \alpha_{z}$ is obtained from the following equations.
$\alpha_{Z}=\alpha_{A}+\varphi_{1}$
$\alpha_{z}=\alpha_{B}+\varphi_{2}$
As can be seen, the centering elements were calculated with the above-mentioned relations without creating a local coordinate system.

## 3. NUMERICAL APPLICATIONS

Here, one numerical application was made for the first method and two numerical applications were made for the second method and classical method.

In the calculations made with the classical method and the second method, the fraction of the integer is taken up to the eighth digit after the comma in order to eliminate the effect of the rounding error on the results (Yüncü and Aslan 2002; Dilaver 2010).

### 3.1. Numerical Application 1

In accordance with Figure 2, calculate the centering elements with the first method, taking into account the measurements made at points A, B and D given in Table 1.

Table 1. Measurements made at points A, B and D

| Station <br> Point | Point of <br> View | Horizontal <br> Angle | Horizontal <br> Distance | Station <br> Point | Point of <br> View | Horizontal <br> Angle | Horizontal <br> Distance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | Z | 0.0000 |  | D | 1 | 0.0000 |  |
|  | D | 0.0000 | 52.535 |  | 2 | 127.3356 |  |
| B | B | 83.3236 | 50.138 |  | $\cdots$ |  |  |
|  | A | 0.0000 | 50.137 |  | B | 242.6565 |  |
|  | D | 60.2750 | 62.526 |  | A | 299.3580 |  |

Solution:

$$
\gamma=\gamma_{\mathrm{Z}}=\gamma_{\mathrm{D}}=83.3236, \quad \beta_{\mathrm{D}}=60.2750, \quad \beta_{\mathrm{Z}}=61.3632,
$$

$\mathrm{AB}=50.138 \mathrm{~m}, \alpha_{A}=299.3580, \alpha_{B}=242.6565$
$\varphi=\gamma+\beta_{D}=143.5986, \quad \psi=200-\left(\gamma+\beta_{z}\right)=55 \mathrm{~s} .3132$
$A Z=\frac{A B * \sin \beta Z,}{\sin \psi}=53.935 \mathrm{~m}$ e $=A Z-A D=1.40 \mathrm{~m}$
$\alpha_{z}=\alpha_{A}+200-400=99.3580, \quad \alpha_{z}=\alpha_{B}-\varphi=99.3579$,

### 3.2. Numerical Application 2

Calculate the centering elements with the classical method and the second method, taking into account the measurements made at points A, B and D given in Table 2, in accordance with Figure 1 and Figure 3.

Average value $\alpha_{z}=99.3580$
Table 2. Measurements made at points A, B and D

| Station <br> Point | Point of <br> View | Horizontal <br> Angle | Horizontal <br> Distance | Station <br> Point | Point of <br> View | Horizontal <br> Angle | Horizontal <br> Distance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | 0.0000 | 94.376 | D | 1 | 0.0000 |  |
|  | Z | 71.8280 |  |  | 2 | 127.3356 |  |
| B | D | 75.4420 |  |  | $\cdots$ |  |  |
|  | Z | 0.0000 |  |  | A | 161.3819 |  |
|  | D | 1.1820 |  |  | B | 244.7059 |  |

Obtaining and simply balancing the elements to be used in the calculations:
$\gamma_{\mathrm{Z}}=71 \mathrm{~g} .8280, \quad \gamma_{\mathrm{D}}=75 \mathrm{~g} .4420, \quad \beta_{\mathrm{D}}=41 \mathrm{~g} .2320, \quad \beta_{\mathrm{z}}=42 \mathrm{~g} .4140$, $A B=94.376 \quad m, \quad \alpha_{A}=161 \mathrm{~g} .3819, \quad \alpha_{B}=244 \mathrm{~g} .7059$, $\omega=244.7059-161.3819=83 \mathrm{~s} .3240$
$\omega+\gamma_{D}+\beta_{D}=199.9980$,
$(83.3240+7 \mathrm{cc})+\left(75.4420+6^{\mathrm{cc}}\right)+\left(41.2320+6^{\mathrm{cc}}\right)=200.0000$, $\gamma_{D^{\prime}}=75.4426, \beta_{D^{\prime}}=41.2326$

### 3.2.1. Solution with the Classical Method

$(A D)=\gamma_{D^{\prime}},(A Z)=\gamma_{Z},(B D)=200-\beta_{D^{\prime}}=158.7674 \quad(B Z)=200-$
$\beta_{\mathrm{z}}=157.5860$
$\mathrm{AD}=\frac{94.376 * \sin \beta_{D}^{\prime}}{\sin \left(\gamma_{D}^{\prime}+\beta_{D}^{\prime}\right)}=58.95139246 \mathrm{~m}$
$\mathrm{BD}=\frac{94.376 * \sin \gamma_{D}^{\prime}}{\sin \left(\gamma_{D}^{\prime}+\beta_{D}^{\prime}\right)}=90.5288754 \mathrm{~m}$
$\mathrm{AZ}=\frac{A B * \sin \beta \mathrm{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}=59.8180741 \mathrm{~m}$
$\mathrm{BZ}=\frac{A B * \sin \gamma_{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}=87.4645758 \mathrm{~m}$
$\mathrm{Y}_{\mathrm{D}}=\mathrm{Y}_{\mathrm{A}}+\mathrm{AD} * \sin (\mathrm{AD})=54.61951034 \mathrm{~m}$,
$\mathrm{X}_{\mathrm{D}}=\mathrm{X}_{\mathrm{A}}+\mathrm{AD}^{*} \cos (\mathrm{AD})=22.18052666 \mathrm{~m}$
$\mathrm{Y}_{\mathrm{Z}}=\mathrm{Y}_{\mathrm{A}}+A Z^{*} \sin (\mathrm{AZ})=54.05600204 \mathrm{~m}$,
$X_{z}=X_{A}+A Z * \cos (A Z)=25.61543739 \mathrm{~m}$
$\Delta Y_{D Z}=-0.5635083, \Delta X_{D Z}=3.43491073$,
(DZ) $=389.64825$,
$\mathrm{DZ}=\sqrt{\left(\Delta Y_{D Z}^{2}+\Delta X_{D Z}^{2}\right)}=3.480826529$
$\varphi=(D Z)-(D A)=114.20565$,
$\psi=(D Z)-(D B)=30.88085$
$\alpha_{Z}=\alpha_{A}+\varphi=275.58755, \alpha_{Z}={ }_{A B}+\psi=275.58675$
Average value $\alpha_{\mathrm{z}}=275.58715$

### 3.2.2. Solution with the Second Method

$\beta_{Z}+\gamma_{Z}=114.2420, \quad \beta_{D}+\gamma_{D}=116.6740, \quad d \gamma=\gamma_{D^{\prime}}-\gamma_{Z}=3.6146$,
$\mathrm{d} \beta=\beta_{\mathrm{Z}}-\beta_{\mathrm{D}^{\prime}}=1.1814$
$\mathrm{b}=\mathrm{AZ}=\frac{A B * \sin \beta \mathrm{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}=59.8180741 \mathrm{~m}$,
$\mathrm{c}=\mathrm{BZ}=\frac{A B * \sin \gamma_{Z},}{\sin \left(\gamma_{\left.Z+\beta_{Z}\right)}\right.}=87.46465758 \mathrm{~m}$
$\mathrm{a}=\mathrm{AD}=\frac{A B * \sin \beta \mathrm{D}^{\prime}}{\sin \left(\beta_{D^{\prime}}+\gamma_{D^{\prime}}\right)}=58.95139246 \mathrm{~mm}$,
$\mathrm{d}=\mathrm{BD}=\frac{A B * \sin \delta \mathrm{D},}{\sin \left(\beta_{D}+\gamma_{D}\right)}=90.5288754 \mathrm{~m}$
$\mathrm{DZ}=\sqrt{\left(a^{2}+b^{2}-2 * a * b * \cos d \gamma\right)}=3.480826518 \mathrm{~m}$
$\mathrm{DZ}=\sqrt{\left(c^{2}+d-2 * c * d * \cos d \beta\right)}=3.48082654 \mathrm{~m}$
$\frac{\varphi_{1}+\psi_{1}}{2}=\frac{200-d \gamma}{2}=98.1927$,
$\frac{\varphi_{1}-\psi_{1}}{2}=\arctan \left((a-b) * \tan \left(\frac{\varphi_{1}+\psi_{1}}{2}\right) /(a+b)=\right.$
-16.01295492
$\varphi_{1}=98.1930+(-16.03904)=82.17974$,
$\psi_{1}=98.1930-(-16.03904)=114.20565$
$\frac{\varphi_{2}+\psi_{2}}{2}=\frac{200-d \beta}{2}=99.4093$,
$\frac{\varphi_{2}-\psi_{2}}{2}=\arctan \left((c-d) * \tan \left(\frac{\varphi_{2}+\psi_{2}}{2}\right) /(c+d)=\right.$
68.52844
$\varphi_{2}=99.4090+(-68.50445)=30.88085, \quad \psi_{2}=99.4090-(-$
$68.50445)==167.9377$
$\alpha_{Z}=\alpha_{A}+\psi_{1}=275.58755$
$\alpha_{Z}=\alpha_{B}+\varphi_{2}=275.58675$
Average value $\alpha_{\mathrm{z}}=275.58715$

### 3.2.3. Numerical Application 3

In accordance with Figure 1 and Figure 3, calculate the centering elements with the classical method and the second method, taking into account the measurements made at points $\mathrm{A}, \mathrm{B}$ and D given in Table 3.

Table 3. Measurements made at points A, B and D

| Station <br> Point | Point of <br> View | Horizontal <br> Angle | Horizontal <br> Distance | Station <br> Point | Point of <br> View | Horizontal <br> Angle | Horizontal <br> Distance |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | 0.0000 | 56.328 | D | 1 | 0.0000 |  |
|  | Z | 58.7462 |  |  | 2 | 138.3164 |  |
| B | D | 60.2465 |  |  | $\cdots$ |  |  |
|  | Z | 0.0000 |  |  | A | 157.1788 |  |
|  | D | 1.3314 |  |  | B | 231.5709 |  |

Obtaining and simply balancing the elements to be used in the calculations:
$\gamma_{\mathrm{Z}}=58 \mathrm{~g} .7462, \quad \gamma_{\mathrm{D}}=60^{\mathrm{g}} .2465, \quad \beta_{\mathrm{D}}=65^{\mathrm{g} .3620}, \quad \beta_{\mathrm{z}}=66 \mathrm{~g} .6934$, $\mathrm{AB}=56.328 \mathrm{~m}, \quad \alpha_{A}=157 \mathrm{~g} .1788, \quad \alpha_{B}=231 \mathrm{~g} .5709$, $\omega=213.5709-157.1788=74 \mathrm{~g} .3921$
$\omega+\gamma_{D}+\beta_{D}=200.0006$,
$\left(74.3921-2^{\mathrm{cc}}\right)+\left(60.2465-2^{\mathrm{cc}}\right)+\left(65.3620-2^{\mathrm{cc}}\right)=200.0000$, $\gamma_{D^{\prime}}=60.2463, \beta_{D^{\prime}}=65.3618$

### 3.3. Solution with the Classical Method

$(A D)=\gamma_{D^{\prime}},(A Z)=\gamma_{Z},(B D)=200-\beta_{D^{\prime}}=134.6382, \quad(B Z)=200-$
$\beta_{\mathrm{z}}=133.3066$
$\mathrm{AD}=\frac{56.328 * \sin \beta_{D}^{\prime}}{\sin \left(\gamma_{D}^{\prime}+\beta_{D}^{\prime}\right)}=52.37442008 \mathrm{~m}$
$\mathrm{BD}=\frac{56.328 * \sin \gamma_{D}^{\prime}}{\sin \left(\gamma_{D}^{\prime}+\beta_{D}^{\prime}\right)}=49.66197913 \mathrm{~m}$
$\mathrm{AZ}=\frac{A B * \sin \beta \mathrm{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}=52.96624644 \mathrm{~m}$
$\mathrm{BZ}=\frac{A B * \sin \gamma_{Z},}{\sin \left(\gamma_{\left.Z+\beta_{Z}\right)}\right.}=48.750235 \mathrm{~m}$
$Y_{D}=Y_{A}+A D^{*} \sin (A D)=42.49058137 m$,
$\mathrm{X}_{\mathrm{D}}=\mathrm{X}_{\mathrm{A}}+\mathrm{AD} * \cos (\mathrm{AD})=30.62075071 \mathrm{~m}$
$\mathrm{Y}_{\mathrm{Z}}=\mathrm{Y}_{\mathrm{A}}+A Z^{*} \sin (\mathrm{AZ})=42.22917398 \mathrm{~m}$,
$\mathrm{X}_{\mathrm{Z}}=\mathrm{X}_{\mathrm{A}}+\mathrm{AZ}^{*} \cos (\mathrm{AZ})=31.97061349 \mathrm{~m}$
$\Delta Y_{D Z}=-0.26140739, \Delta X_{D Z}=1.34986278,(D Z)=387.8223$,
$\mathrm{DZ}=\sqrt{\left(\Delta Y_{D Z}^{2}+\Delta X_{D Z}^{2}\right)}=1.374941216$
$\varphi=(D Z)-(D A)=127.5760, \quad \psi=(D Z)-(D B)=53.1841$
$\alpha_{z}=\alpha_{A}+\varphi=284.7548$,
$\alpha_{Z}=\alpha_{A B}+\psi=284.7550$
Mean value $\alpha_{z}=284.7549$

### 3.3.1. Solution with the Second Method

$\mathrm{a}=\mathrm{AD}=\frac{56.328 * \sin \beta_{D}^{\prime}}{\sin \left(\gamma_{D}^{\prime}+\beta_{D}^{\prime}\right)}=52.37442008 \mathrm{~m}$
$\mathrm{d}=\mathrm{BD}=\frac{56.328 * \sin \gamma_{D}^{\prime}}{\sin \left(\gamma_{D}^{\prime}+\beta_{D}^{\prime}\right)}=49.66197913 \mathrm{~m}$

$$
\begin{aligned}
& \mathrm{b}=\mathrm{AZ}=\frac{A B * \sin \beta \mathrm{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}=52.96624644 \mathrm{~m} \\
& \mathrm{c}=\mathrm{BZ}=\frac{A B * \sin \gamma_{Z},}{\sin \left(\gamma_{Z+\beta_{Z}}\right)}=48.750235 \mathrm{~m} \\
& \mathrm{~d} \gamma=\gamma_{\mathrm{D}^{\prime}-\gamma \mathrm{Z}}=1.5001, \mathrm{~d} \beta=\beta_{\mathrm{Z}}-\beta_{\mathrm{D}^{\prime}}=1.3316 \\
& \mathrm{DZ}=\sqrt{\left(a^{2}+b^{2}-2 * a * b * \cos d \gamma\right)}=1.374941228 \mathrm{~m} \\
& \mathrm{DZ}=\sqrt{\left(c^{2}+d-2 * c * d * \cos d \beta\right)}=1.374941188 \mathrm{~m} \\
& \frac{\varphi_{1}+\psi_{1}}{2}=\frac{200-d \gamma}{2}=99.24995, \\
& \frac{\varphi_{1}-\psi_{1}}{2}=\arctan \left((a-b) * \tan \left(\frac{\varphi_{1}+\psi_{1}}{2}\right) /(a+b)=\right.
\end{aligned}
$$

$$
28.32604
$$

$$
\varphi_{1}=98.1930+(-16.03904)=70.9239,
$$

$$
\psi_{1}=98.1930-(-16.03904)=127.5760
$$

$$
\frac{\varphi_{2}+\psi_{2}}{2}=\frac{200-d \beta}{2}=99.3342
$$

$$
\frac{\varphi_{2}-\psi_{2}}{2}=\arctan \left((c-d) * \tan \left(\frac{\varphi_{2}+\psi_{2}}{2}\right) /(c+d)=\right.
$$

46.150109
$\varphi_{2}=99.4090+(-68.50445)=53.18409, \quad \psi_{2}=99.4090-(-$ 68.50445) $==145.484309$

$$
\alpha_{Z}=\alpha_{A}+\psi_{1}=284.7548 \quad \alpha_{Z}=\alpha_{B}+\varphi_{2}=284.7550
$$

Mean value $\alpha \mathrm{Z}=284.7549$

## 4. EXAMINATION OF NUMERICAL APPLICATION RESULTS

When the results of the centering elements obtained in the above 3.2 and 3.3 numerical applications (Table 4) are examined; It was observed that there was no difference between the elements calculated with the classical method and the second method.

Table 4. Summary of the results of 3.2 and 3.3 numerical applications according to both method

| Numerical <br> Application <br> Number | Method Used | Calculation Results of Numerical Applications |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | DZ | $\alpha_{Z}=\alpha_{A}+\varphi$ | $\alpha_{Z}=\alpha_{B}+\psi$ | $\alpha_{z}=\alpha_{A}+\psi_{1}$ | $\alpha_{Z}=\alpha_{B}+\varphi_{2}$ |
| 2.2 | Classic | 3.480826 | 275.58755 | 275.58675 |  |  |
|  | Classic | 3.480826 |  |  | 275.58755 | 275.58675 |
| 2.3 | Classic | 1.374941 | 284.7548 | 284.7549 |  |  |
|  | Second | 1.374941 |  |  | 284.7548 | 284.7549 |

## 5. RESULTS

In the calculation of the centering elements, it is recommended to apply the first method in order to be established from the calculation load. With the electronic tachometer produced today, it is possible to look from one point to another with a certain directional angle. Taking advantage of this feature of the instrument, it is recommended to look at the off-center point, the auxiliary point that enables this point to be marked on the minaret balcony, with a directional angle of

200 g .0000 for ease of calculation in the first method. In this case, the $\alpha_{z}$ centering element is 0 g. 0000 . Numerical applications of centering elements with the classical method and the second method; It was seen that there was no difference between the calculation results and the results were equal to each other. The advantage of the second method is that it does not require coordinate calculation and it can be said that it is equivalent to the classical method in terms of processing load.

## Author contributions

The contributions of Yazar1, Yazar2, Yazar3 and Yazar4 to this article is equal.

## Conflicts of interest

The authors declare no conflicts of interest.

## Statement of Research and Publication Ethics

The authors declare that this study complies with Research and Publication Ethics

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