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Identifying the best fitting 3D deformation model using information criteria

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ABSTRACT

3D deformation studies are usually based on 12-parameter affine transformation model. Deformation part of this model is expressed with three scale factors and three skew parameters along x, y, z axes. However, actual deformation of the monitored object may have a different structure than the one prescribed by this model. For instance, there may exist skewness along only xy plane, or one dilation along only z-axis. In this sense, we encounter with possible fifteen different deformation models such as 7-parameter (similarity), 8-parameter affine, 10-parameter affine, etc. The question arising is which one fits best to the coordinates. For this aim, we use Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The efficiencies of these criteria are studied within many deformation examples using Monte-Carlo simulations. According to the numerical examples, both criteria can detect the true model successfully with a success rate ranging from 53 to 97% if the deformation parameter is three times bigger than its standard error.

1. INTRODUCTION

Terrestrial, aerial and space methods today yield continuous and (near) real-time 3D spatial data with unprecedented precision and resolution thanks to the advanced measuring sensors. This high-quality data allows us to monitor the earthbound bodies (the Earth crust or engineering structures) and analyze their deformations continuously (or discretely) in 3D space.

The temporal (Cartesian) coordinate changes of the points characterizing the monitored body consist of relative and non-relative parts. The relative part deals with translation and rotation of the body whereas the non-relative part corresponds to the shape and size change (deformation) of the body in time (Aydin 2017). To discriminate two parts against each other, the following 3D strain model is applied to the coordinate differences (**d**) monitored between two epochs of time:

$$\mathbf{d}=\mathbf{y}-\mathbf{x}=\mathbf{t}+\mathbf{E}\mathbf{x}$$

where **x** and **y** denote the coordinate vectors in the initial and present epochs, respectively; **t** denotes the translation, and **E** is the anti-symmetric strain tensor including nine different elements. With these strain elements, can define the dilation, shear, differential rotation etc. We may also consider 12-parameter affine transformation between **x** and **y** coordinates (Amiri-Simkooei 2018):

$$\mathbf{y}=\mathbf{t}+\mathbf{RLMx}$$

where **R** is the skew-symmetric differential rotation matrix; **L** is the diagonal scale factor matrix, and **M** is the matrix of skew parameters. If we subtract the initial coordinates "**x**" from both sides of this affine model, we obtain "**E=RLM-I**", where **I** denotes the identity matrix. Therefore, 3D strain model is a 12-parameter affine transformation model applied between initial and present epoch coordinates.

In deformation analysis, we deal with the scale matrix **L** and the skew matrix **M**. In the above-given affine model, they represent three scale factors along x, y, z axes and three skew parameters along each pair of axes, namely between x-y, x-z and y-z axes (Note that scale factors correspond the dilations while skew parameters correspond to the shears in continuum mechanics). However, actual deformation model may be different from the one in the prescribed model. For instance, no skewness may exist in the xz plane while only x axis has a significant scale change. In such a combinatorial way, we may consider fifteen different deformation models,

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which are summarized in Table 1. Each of them includes common translations and rotations but has different scale factors and skew parameters.

Table 1. Models and their number of parameters (u) (Note: Shaded elements exist in the model; S=Similarity, A=Affine; [...]=groups)

Model (u)	Scale			Skew		
	x	y	z	x-y	x-z	y-z
1-S (7)	■	■ Common	■			
2-A (7)	■					
3-A (7) [i]		■				
4-A (7)			■			
5-A (8)		■	■			
6-A (8) [ii]	■		■			
7-A (8)	■	■				
8-A (9)	■	■	■			
9-A (10)	■	■	■			■
10-A (10) [iii]	■	■	■		■	
11-A (10)	■	■	■	■		
12-A (11)	■	■	■		■	■
13-A (11) [iv]	■	■	■	■		■
14-A (11)	■	■	■	■	■	
15-A (12)	■	■	■	■	■	■

The question we mainly pose here is how to identify the best fitting model. Conventional model tests with Fisher-distribution (Demirel 2005) may not be useful to obtain the best model among the other possible models since this test compares only two models at a time. We, therefore, consider information criteria, which are easily adapted to our potential models as shown in (Lehmann 2014) and (Even-Tzur 2020) for coordinate transformations. Two criteria, called Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), are considered to find out the best 3D deformation model. The "Method Section" briefly expresses these criteria. To investigate how they can identify the "true" model successfully, we use many deformation examples generated by a Monte-Carlo simulation strategy. The success rates are given in the "Results Section". In the last two sections, we discuss the results and concludes our study.

2. METHOD

2.1. AIC and BIC for the 3D Deformation Models

The previously given 12-parameter affine model deals with one object point. Using the same notation "**y**" and "**x**" for the coordinate vectors of all points in the deformation study, we give the linearized observation equations as follows:

$$\mathbf{y}-\mathbf{x}-(\mathbf{e}_y-\mathbf{e}_x)=\mathbf{A}\mathbf{c}$$

where **e** denotes the error vector of the coordinates; **A** is the design matrix and **c** are the vector of unknown u=12 parameters (translations, differential rotations, scale factor changes and skew parameters). Note that the least-squares method applied to this mathematical model yields the equivalent results with those from total-least squares solution (e.g. Uygur et al. 2020) since the **RLM** in the second equation goes to identity matrix in deformation studies, and the noises in the coordinates are relatively too low. Expressing the error differences with "$\mathbf{e}=(\mathbf{e}_y-\mathbf{e}_x)$", we obtain the quadratic form of the 12-parameter affine model as "$\Omega=\mathbf{e}^T\mathbf{P}\mathbf{e}$", where **P** is the weight matrix set based on the known cofactor matrix of the coordinate difference "**y**–**x**". The AIC and BIC associated with the model then can be obtained by (Even-Tzur 2020):

$$AIC=3p\times\ln(\Omega)+2u\times3p/(3p-u-1)$$
$$BIC=3p\times\ln(\Omega/3p)+u\times\ln(3p)$$

where p is the number of points.

The AIC and BIC of each potential model in Table 1 can be obtained with the same methodology but using the corresponding design matrix **A** and the number of parameters u associated with the model (Note: For saving space, we do not give the design matrices of the models explicitly here. The readers can derive this matrix from (Amiri-Simkooei 2018) easily or can contact the corresponding author of our study). Finally, the model which gives the minimum AIC (or BIC) is accepted as the best model describing the deformation structure of the investigated body.

2.2. Generating Deformed Bodies

In order to investigate how AIC (or BIC) is successful in identifying the best fitting model, we need different examples in which we know which model is the true one. A "true" model can be known beforehand if and only if we generate these examples ourselves by a Monte-Carlo simulation strategy. For this aim, we follow the strategy given in (Uygur et al. 2020). Our random simulation consists of the following steps:

- The x, y and z coordinates of the object points, of which numbers were selected randomly between 7 and 15, were generated between 10^3 and 10^5 m locating the points into the proper grids. With these grids, a proper constellation without any bad condition is guaranteed.
- The coordinates generated now correspond to the "errorless" coordinates in the initial epoch. Later on, the fully populated and positive definite covariance matrices of the initial and present epochs were produced. The standard errors of the parameters were obtained having applied an approximate transformation using the epoch covariance matrices.
- Zero-translations and rotations were assumed. The scale factors and skew parameters were generated randomly by

  Scale Factor=1+τ×(its standard error)
  Skew Parameter=τ×(its standard error)

  where τ is a random number ranging from 2 to 6. The sign of the amplitude τ was also randomly selected.
- The present epoch "errorless" coordinates were evaluated using the "true" model based on the above-given scales and skewness. In such a way, the

deformation is being incorporated into the coordinates. Afterward, the random errors, normally distributed according to the previously mentioned covariance matrices, were added to the initial and present epoch "errorless" coordinates to get the vectors **x** and **y**. This set of pairs represent one random sample.

2.3. Mean Success Rate (MSR)

For each "true" model, we produced m=1000 random samples. Each random sample was solved by each model in Table 1. The number of samples (k) in which the corresponding model was identified as the best fitting model based on the AIC (or BIC) was counted to get the following MSR (Hekimoglu and Koch 1999):

MSR=(k/m)×100

3. RESULTS

Totally 15 MSRs were evaluated for each "true" model. It is expected that the MSR associated with the "true model" goes to 100%. However, statistically speaking, it is not possible to have this probability. The MSRs for different cases are given in this section.

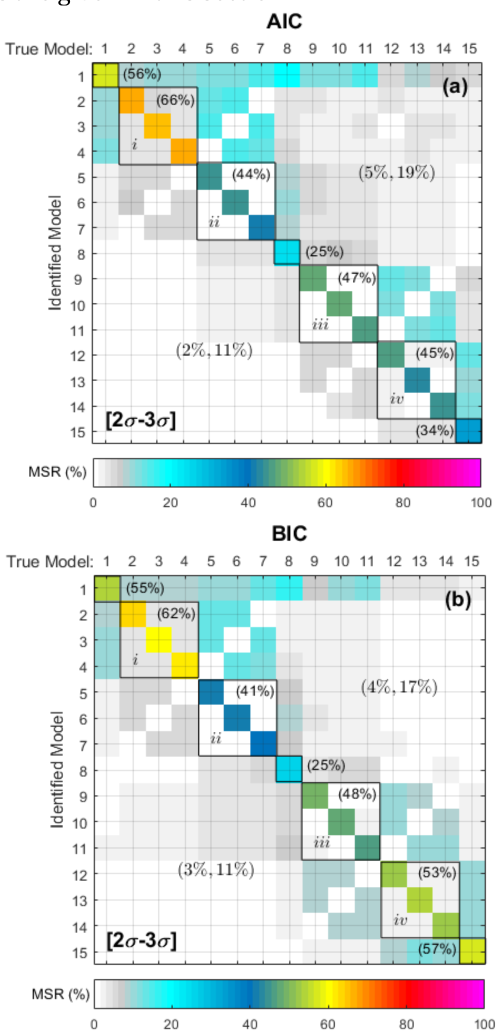

Figure 1. MSRs for fifteen models by using a) AIC and b) BIC while the deformation range is 2σ-3σ

Successfully identification of the true model relies on the magnitude of the scale change and skew parameters. Firstly, the amplitude τ was selected between 2 and 3 such that the deformation parameters lie in the range of 2σ-3σ (σ=stands for the standard error of the corresponding parameter). The obtained MSRs for this range are shown as a color matrix in Figure 1.

The diagonal elements of this matrix in Figure 1 refer to the successfully identifying the true model while the off-diagonals refer to the wrongly identification (the percentage values over the off-diagonals denote the minimum and maximum MSRs of the incorrect results). In both information criteria, the most successful results are obtained for the models in group-i (Affine-7) whereas the least successful results are obtained for the Affine-9 model (MSR is 25%). The BIC is more successful (MSR=57%) than the AIC (MSR=34%) if the true model is the Affine-12 model. However, in general, there is no other significant difference between AIC and BIC since the range of 2σ-3σ is too low as is known from deformation and outlier detection studies. We, therefore, increase the range to 3σ-6σ, and repeat the above-expressed procedure. The evaluated MSRs are shown in Figure 2. We discuss these results in the next section.

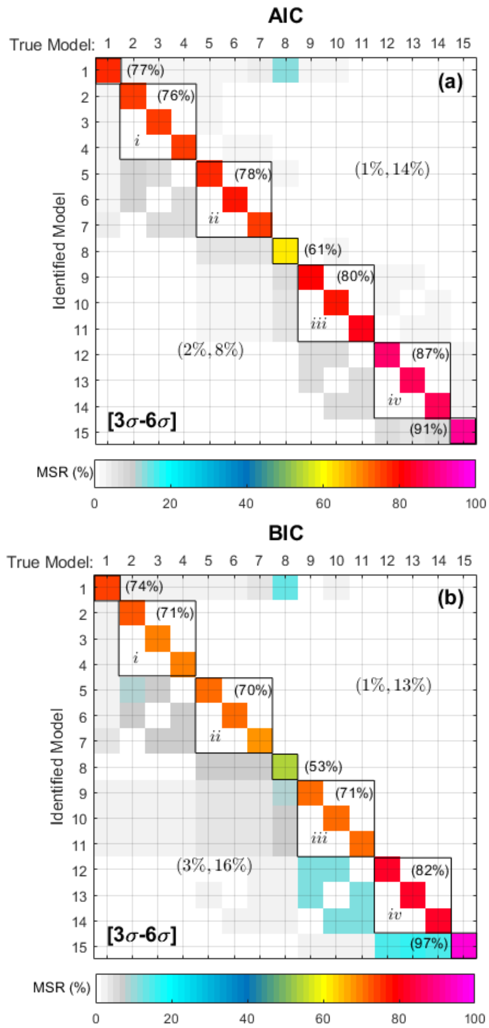

Figure 2. MSRs for fifteen models by using a) AIC and b) BIC while the deformation range is 3σ-6σ

4. DISCUSSION

According to the results in Figure 2, we see that the AIC provides more successful identification of the true model except for the Affine-12 deformation model. The BIC can identify the Affine-12 model with the success rate of 97%. The MSRs range from 61% to 91% (79% on average) and 53% to 97% (74% on average) using AIC and BIC, respectively. Both criteria fail for the Affine-9 model, which actually correspond to the rigid body volume change. This is because the Affine-9 model is usually wrongly identified as the Similarity-7 model. Hence, it may not be easy to discriminate the Affine-9 model against the Similarity-7 model.

Furthermore, we repeat our analysis using geodetic coordinates. Similar results are also valid for these geodetic constellations. Due to lack of space, however, we left these analyses to our next studies.

Correspondingly, although there are some minor differences between the AIC and BIC in different deformation models, we can interpret that one may identify the true deformation model with a success rate of about 75% using both criteria if the deformation parameters are 3 times bigger than their standard errors. This result is important for practical studies seeking for the best fitting deformation model among many different models.

5. CONCLUSION

We study the AIC and BIC in identifying the best fitting 3D deformation model. For this aim, fifteen different deformation models are considered.

In order to investigate how these criteria are successful in identifying the true deformation model among the potential fifteen models, we adapt the MSR concept. For this aim, we use the simulated random samples by considering a Monte-Carlo simulation strategy.

According to the numerical results, both criteria may fail in identifying the true model if the deformation parameters lie between 2σ and 3σ. They can identify the true model with a reasonable probability if the range is between 3σ and 6σ. This is not a surprising result because the "3σ" magnitude refers also to the minimum detectable outlier and deformation with 80% test power as is known from geodetic studies. For this range, we see that the AIC and BIC can identify the true model with the success rates of about 79% and 74%, respectively. The type of the actual deformation model affects these statistics. For instance, the worst results belong to the Affine-9 deformation model which generally is confused with the Similarity-7 model by the criteria. However, it is worth mentioning that these bad results may come from the random samples since we do not take into account how much dilation differences between x, y and z axes should be involved in the simulation strategy. This issue is dealing with deriving the minimum identifiable (detectable) deformation parameter by using the information criteria, which needs a more detailed statistical investigation.

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